Understanding Transmission Zero Movement In Cross-Coupled Filters

Robert J. Wenzel

Wenzel/Erlinger Associates, Inc., a Subsidiary of Allen Telecom, Inc., Agoura Hills, CA 91301

Abstract — **Rules for transmission zero movement in real circuit cross-coupled filters and a classification of two primary transmission zero types are given. The rules and classifications apply to filters of arbitrary bandwidth and give insight into response skewing and "disappearing" zeros. Transmission zero sensitivity and performance limitations are described and related to circuit topology and coupling element types.**

I. Introduction

The use of coupling between non-adjacent resonators (cross-coupling) in bandpass filters is well established, and virtually every combination of inter-resonator couplings has been investigated. The tuning of such filters and an explanation of why a filter response behaves the way it does can appear to be quite confusing. This is especially true when comparing cross-coupled designs with cascade realizations for which independent control of each transmission zero is usually obvious from the circuit topology. The situation becomes even more difficult to understand as filter bandwidth increases. Narrowband prototype circuits no longer accurately describe how real circuits will perform and inaccuracies caused by use of such prototype circuits can be substantial, even at relatively narrow bandwidths.

II. Transmission Zero Movement Rules and Classification of Zeros

The behavior of arbitrary bandwidth cross-coupled filters for which we have a realistic circuit model can be investigated and better understood by considering how circuit transmission zeros move as elements are perturbed. A definition of a perturbed element can include the creation of an element that did not previously exist, i.e. it's value having been perturbed from zero or infinite value. Transmission zero movement in real circuits obey the following three rules:

RULE 1. All transmission zeros occur with quadrantal symmetry in the complex plane.

RULE 2. Perturbing an element causes transmission zeros to move in continuous paths, i.e. they never jump to a new location.

RULE 3. Perturbing an element never causes related transmission zeros to follow one another, i.e. they move closer together or further apart, but not in the same direction.

In addition to the above rules of movement, transmission zeros can be of two movement types:

Type 1. Zeros restricted to real frequencies.

Type 2. Zeros that can be at real or complex frequencies.

In any given circuit, the zero movement type is determined by the circuit topology and specific circuit elements.

III. Examples

For illustration, consider the four resonator inductively coupled bandpass filter circuit of Fig. 1(a). This circuit has eight transmission zeros, seven at ω = infinity and one at $\omega = 0$. The simplest Type 1 real frequency zero occurs if we place a capacitor in parallel with any of the coupling inductors as shown in Fig. 1(b) giving a cascade realization. This zero is locked on the real frequency axis, and cannot move off the axis with perturbation of existing finite elements. Next consider adding an inductor from resonator 1 to 3 to the circuit of Fig. 1(a) giving the circuit of Fig. 1(c). This creates the well known high side triplet, and gives a Type 1 transmission zero above the passband. We know it is a Type 1 transmission zero by examining the transmission zero structure before and after the element was added (perturbed). We started with seven at infinity and one at zero. The circuit of Fig. 1(c) has five at infinity and one at zero leaving just two to be accounted for. By Rule 1 the zeros must be either on the sigma axis or the $j\omega$ axis. By analysis or the fact that we know that the circuit can produce a high side zero, the zeros must be on the $j\omega$ axis. As with the cascade circuit, the zeros are

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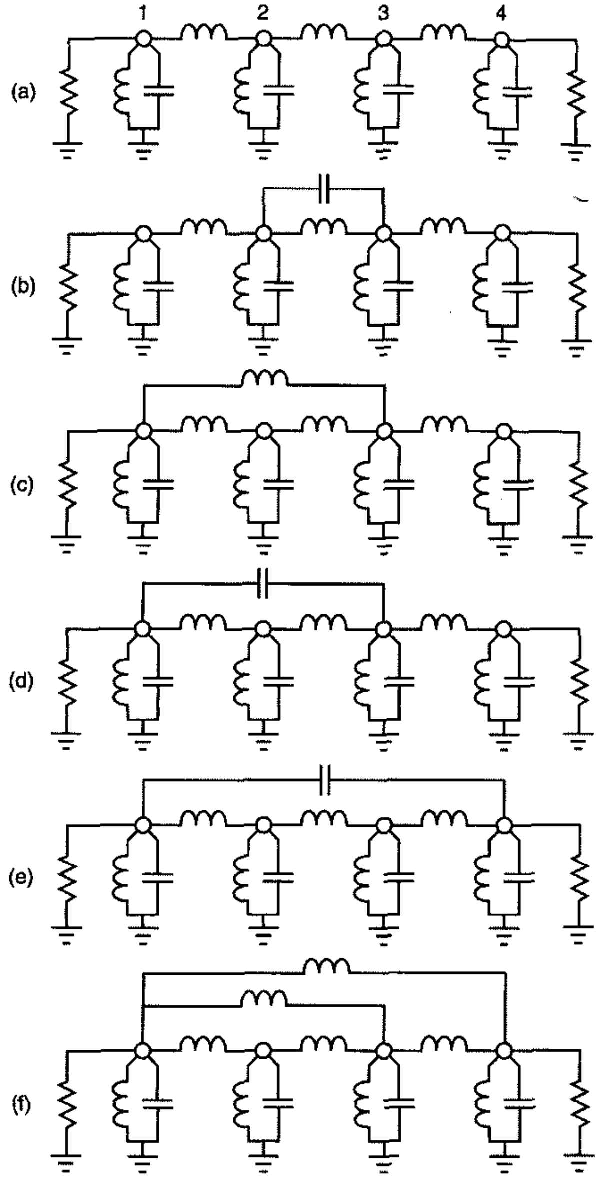

Fig. 1. Four resonator inductively coupled bandpass filter example circuits.

locked on the jω axis independent of perturbation of any finite existing element.

Next consider adding a capacitor from resonator 1 to 3 to the circuit of Fig. 1(a) giving the circuit of Fig. 1(d). This is the well known low side triplet circuit. These zeros are of Type 2, and may or may not appear at real frequencies. Examining transmission zeros, we started with seven at infinity and one at zero. Adding the bridging capacitor we have three at infinity and one at zero leaving four un-

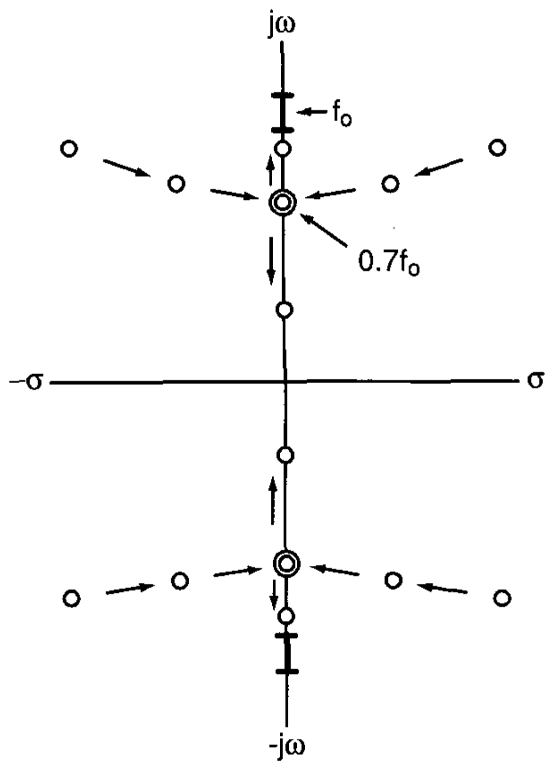

Fig. 2. Movement of transmission zeroes for the circuit in Fig. 1(d). The zeroes move in from infinity, reach the jω axis then split into two zeros that move on the jω axis according to Rule 3.

accounted for. From Rules 1-3 and the fact that we know that the circuit can have a low side transmission zero, the only possibility is that the zeros break from infinity as a complex quadruplet. For small values of the bridging capacitor, no transmission zero will appear at any real frequency. As the value of the bridging capacitor is increased, and other elements are modified to maintain say equal ripple response, the transmission zeros move as shown in Fig. 2, ultimately reaching the jω axis, and then splitting to produce two zeros that move on the jω axis in accordance with Rule 3. From a study of the above movement, it has been found that the frequency at which the two zeros appear doubled up on the jω axis is relatively independent of filter design parameters such as ripple value and bandwidth. For a lumped element circuit this "break frequency" (BF) is approximately $0.7f_0$, where f_0 is the filter center frequency. As filter bandwidth is increased, this near constant BF causes the achievable stopband lobe level to decrease. Stopband rejection is highly sensitive to filter tuning whenever the transmission zeros are near the BF. With narrowband filters, the BF for a lowside transmission zero is quite far from the passband, and the transmission zeros

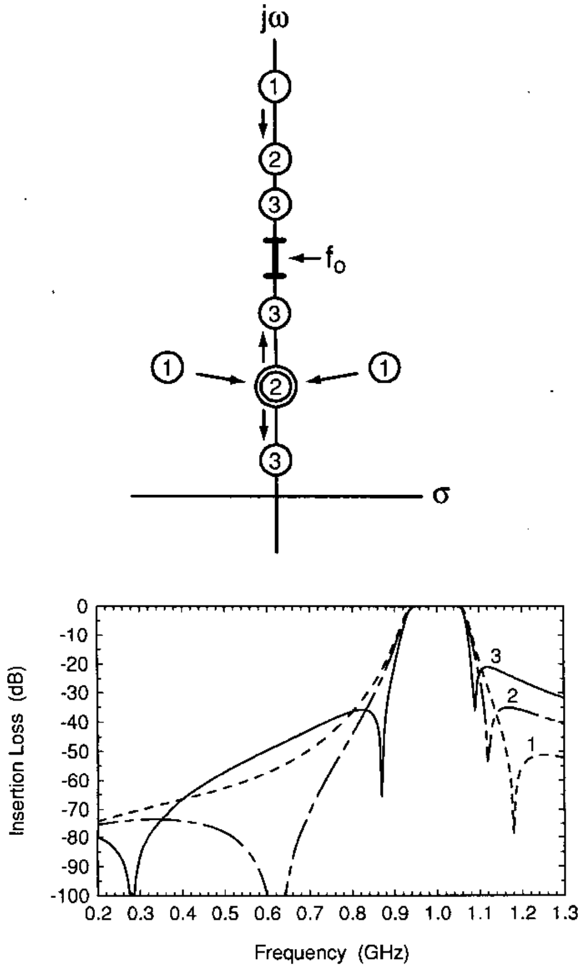

Fig. 3. Movement of transmission zeros for the well known quad circuit, Fig. 1(e), and the resulting insertion loss plots for the three noted transition zero locations.

are often widely split such that the second lowside zero is far below the close in zero.

Next consider the circuit of Fig. 1(a) in which we add a capacitor from resonator 1 to 4 as shown in Fig. 1(e). This is the well known quad circuit used to produce a transmission zero on both sides of the passband. Counting transmission zeros, after adding the bridging capacitor we have one at ω = infinity and one at $\omega = 0$ and six unaccounted for. When the capacitor is added, a Type 1 zero appears on the $j\omega$ axis above the passband, and a Type 2 quadruplet appears in the complex plane below the passband. As the bridging capacitor is increased in value (and the other elements are adjusted to give say equal ripple passband response), the Type 1 zero moves down the $j\omega$ axis toward the passband, and the Type 2 quadruplet moves toward the

TABLE I
SUMMARY OF SAMPLE LOBE LEVELS

BW	Break Frequency (GHz)	Upper Transmission Zero (GHz)	Low Lobe Level	High Lobe Level
1%	0.624	1.114	-156dB	-114dB
2%	0.624	1.114	-132dB	-90dB
5%	0.624	1.116	-98dB	-59dB
10%	0.623	1.122	-74dB	-35dB
20%	0.615	1.145	-50dB	-13.5dB
30%	0.597	1.179	-38dB	-4.7dB

$j\omega$ axis, ultimately reaching the axis at the BF. The low side pair then split as in the case of the low side triplet. The movement of the transmission zeros and the corresponding insertion loss response is shown in Fig. 3. As with the triplet low side zeros, the BF for the quad circuit has been found to be relatively independent of filter bandwidth and ripple value. The low side zeros are also highly sensitive to element tuning when they are near the BF. The frequency location of the high side zero when the low side zeros are at the BF is also relatively independent of filter bandwidth and ripple value, and is much closer to the passband than is the BF. The above behavior limits the stopband lobe levels that can be obtained with transmission zeros appearing on both sides of the passband. It also results in substantial skewing of the response as the low side BF and corresponding upper stopband zero are highly skewed about f_0.

The above results also hold for the case of distributed resonators with lumped or distributed loading. For example, with 45 degree combline resonators at 1GHz, lumped capacitor loading, and a 26dB, N = 4 equal ripple return loss response, the low side BF is at about 0.624GHz when the high side zero is at about 1.12GHz, relatively independent of filter bandwidth and ripple value. Obviously for say 30% bandwidth, the upper zero must be above 1.12GHz for equal ripple passband response, but even at 30% bandwidth the lowside BF is at about 0.6GHz and the upper zero is at 1.18GHz. The stopband lobe levels for the above 30% bandwidth case when the lowside zeros are at the BF are 38dB in the lower lobe and 4.7dB in the upper lobe. Table I gives a summary of sample lobe levels achievable for the above N = 4 case versus bandwidth when the low side zeros are at the BF.

As more non-adjacent resonator couplings are added to filter circuits we can gain more real frequency transmission zeros. However such zeros tend to be Type 2 zeros rather than Type 1 zeros, and are thus more sensitive to filter tuning. As an example, assume that we add inductive coupling from resonators 1 to 3 and from 1 to 4 to the cir-

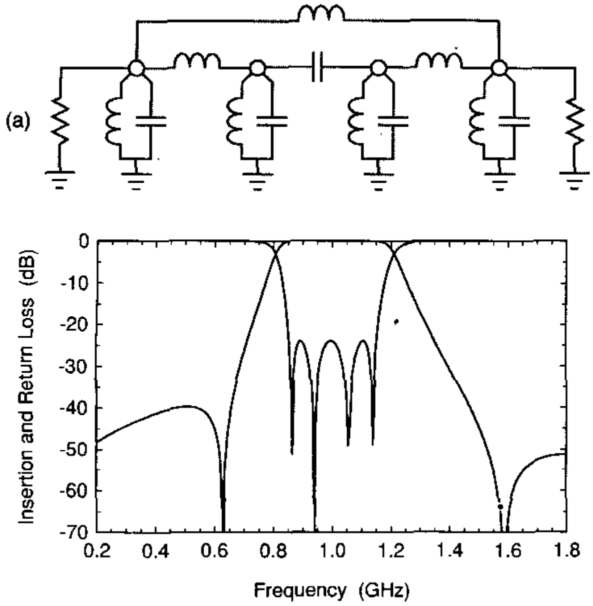

Fig. 4. Quad circuit with capacitive mainline coupling (a) and insertion loss / return loss responses (b).

cuit of Fig. 1(a) giving what we will call the "complex quad" circuit of Fig. 1(f). This gives three zeros at $\omega =$ infinity, one zero at $\omega = 0$, and four unaccounted for. From a previous example we know that if L14 was infinite and L13 was finite, we would have a Type 1 zero on the $j\omega$ axis. As L14 becomes finite, from Rule 1, the zeros must maintain quadrantal symmetry. From Rule 2 the required four zeros cannot jump to a complex plane quadruplet. Thus from Rule 3, a new zero must appear on the $j\omega$ axis and move toward the passband while the existing close in zero must move away from the passband toward the new zero. As we adjust the values of L13 and L14 (along with other elements to say maintain equal ripple performance), we can place the two zeros at any desired frequencies. In narrow band filters, we tend to place these zeros close to the passband and often close to each other. If we place them at the same frequency we find that we are at the BF for a complex quad of zeros. At this point, slight resonator tuning or coupling adjustment can cause the zeros to split on the axis, or go into the complex plane and "disappear." The "complex quad" circuit of Fig. 1(f) is often used with transmission zeros that are near the BF. It tends to be more sensitive, even for very narrowband applications, than triplet and simple quad circuits which have Type 1 zeros and / or Type 2 zeros that are widely split in frequency (far from the BF).

As a final example, assume we would like to realize a cross-coupled 30% bandwidth simple quad with transmission zeros on both sides of the passband and a stopband lobe level of -40 dB. The inductive coupled configuration of Fig. 1(e) is useless for such a design. To get a practical solution, consider the circuit of Fig. 4(a) in which we simply change the 2-3 inductive coupling to capacitive, and the 1-4 capacitive coupling to inductive. This circuit has three zeros at $\omega =$ infinity, one at $\omega =$ zero, and two Type 1 zeros, one above and one below the passband. These zeros are locked on the $j\omega$ axis and will always be present independent of existing element perturbation. The response of a lumped element design is shown in Fig. 4(b). While not perfectly symmetric, the response is quite respectable, of relatively low sensitivity, and far superior to the more common simple quad circuit of Fig. 1(e).

IV. Conclusion

The behavior of real circuit cross-coupled filters of arbitrary bandwidth has been described in terms of transmission zero location and movement. Rules of transmission zero movement have been given, and transmission zeros have been classified as being of two distinct movement types. The techniques described have been illustrated by means of simple well known triplet and quad circuits. However, these techniques can be applied to much more complex circuits, and are useful for understanding performance limitations and tuning sensitivity of real circuits. Improved performance by proper choice of coupling element type was also illustrated, and is of increasing importance as filter bandwidth increases.